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Private road networks with uncertain demand

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ABSTRACT

We study the efficiency of private supply of roads under demand uncertainty and evaluate various regulatory policies. Due to demand uncertainty, capacity is decided before demand is known but tolls can be adjusted after demand is known. Policy implications can differ from those under deterministic demand. For instance, for serial links, the toll in the second-best zero-profit case is no longer equal to the marginal external congestion cost. In the first-best scenario, the capacity under uncertain demand is higher than that under deterministic demand of the same expected value, though self-financing still holds in expected terms. Regulation by perfect competitive auction cannot replicate the second-best zero-profit result and thus leads to a lower welfare, whereas without uncertainty, various forms of competitive auctions can attain this second-best optimum. For more complex networks, when private firms add capacity in turn, contrary to the case without demand uncertainty, some forms of auction perform better than others with demand uncertainty.

1. Introduction

There has been wide and growing interest in the private supply of roads, in addition to the public supply, as a solution to increasing traffic congestion. Various public–private partnerships and Build–Operate–Transfer projects have taken place around the world, for example in Chile, Colombia, Mexico, China, and the U.S.A. (Fisher and Babbar (1996)). But even in the E.U., around 30% of motorways are ‘private’ (Verhoef (2008)). Often cited reasons are that governments in many countries have insufficient public funds to finance new road projects and private firms may work more efficiently than the public sector. A disadvantage is that private suppliers have market power and tend to maximize their profits, resulting in a loss of social welfare compared to optimal pricing. The trade-offs have been studied by numerous scholars: starting with a debate between Pigou (1920) and Knight (1924) considering a simple network, to De Vany and Saving (1980) and de Palma (1992) on competing private toll roads, to de Palma and Lindsey (2000) on dynamic congestion and private roads, and Wang, Lindsey, and Yang (2011) on nonlinear pricing on private roads. Researchers have proposed several regulatory policies to overcome the disadvantages of the private supply of roads. But they usually overlook a prevailing phenomenon in road construction: capacity is set when future demand is still uncertain and can only be estimated, even though tolls can be subsequently adjusted according to the realized demand.

Travel flows and demand for road use are difficult to predict and very uncertain. According to Flyvbjerg, Skamris Holm, and Buhl (2006), actual traffic deviates from that forecast during the planning stage by more than 20% for half of the roads projects investigated. Uncertainty is very important for private suppliers of roads: construction costs are sunk from the start, and uncertainty means that there is a large risk of running large losses and even going bankrupt. Moreover, firms may have to pay large risk premiums on loans used to finance the road construction (Gómez-Ibáñez and Meyer (2011) and Engel, Fischer, and Galetovic (2001)). Conversely, the government is less affected by a single road making a loss. Our analysis can be applied to long-term demand uncertainty due to unpredictable economic booms and busts over the years, but also to short-term demand fluctuations, such as predictable peak and non-peak hours alternating during the day. The core assumption is that capacities are set first and hard to change, but prices can be set later and are more adjustable to suit actual demand situations. Most papers on private roads and BOT projects have not included demand uncertainty, whereas, as we will show, demand uncertainty certainly affects the outcome of settings where private firms are free to build new roads and where they are regulated by a tender auction.

Policy makers and practitioners alike can benefit from a better understanding of the impact of demand uncertainty on equilibrium tolls, capacities, profits, and consumer surplus. We make the conventional assumptions that private firms maximize profit, while the public sector

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maximizes social welfare.¹ We study both simple and complex networks. The advantage of studying simple networks is to have clear economic explanations and interpretations, which can form a basis for rules of thumb in policy making. The economic insights gained from a small network usually carry over to more complex networks. We also develop simulation methods to examine larger networks. For simple networks, we distinguish between parallel links and serial links, and analytically study various scenarios that are used as benchmarks to assess the efficiency of private supply. We also examine, in terms of the impact on social welfare, various competitive auctions to regulate the market. For more complex networks, we are interested in the development of the private supply of road networks over time through competition. We run simulations to compare the equilibrium outcomes of free entry versus entry by regulation.

Our study is related to two strands of the literature. The first one examines the effects of the private supply of roads in a mixed network without demand uncertainty. Following Mohring and Harwitz (1962), Yang and Meng (2002) show that if both the toll and the capacity are set optimally for every link and neutral scale economies prevail, the Pigouvian toll is optimal and each road is self-financing: the collected tolls cover the capacity costs. Verhoef (2007) demonstrates that when there is an untolled substitute road in the network, a road supplier who prices and invests second-best optimally makes a loss, so a subsidy from the government is needed to achieve the desired second-best social welfare level. If such a subsidy is ruled out, Verhoef (2008) derives the highest social welfare under the condition that the private firm makes at least zero profit. This is a natural benchmark with which to compare the efficiency of various alternative ways of regulating the private supply of roads through competitive auctions or free entry, since competitive auctions also drive the profits of the winning bid to zero. The optimal toll in the second-best zero-profit case is again Pigouvian for both serial and parallel links. In addition, Verhoef (2008), Ubbels and Verhoef (2008) and Van den Berg (2013) found that among many possible regulatory schemes, two competitive auctions, namely the “patronage auction” and the “generalized price auction,” are preferable, as they make the private firm choose the socially optimal tolls and capacity under the zero-profit condition. However, when we study these two auctions in the case with uncertain demand, we find that they can no longer achieve the social optimum.

The second strand of the literature considers a single road with demand uncertainty. De Vany and Saving (1977), Kraus (1982), D’Ouille and McDonald (1990), Arnott, de Palma, and Lindsey (1996) and Lu and Meng (2017a) all study different forms of demand uncertainty for a single link, and find that in general a larger investment in capacity is justified, because the optimal capacity with demand uncertainty is larger than the optimal capacity for a deterministic demand with the same expected value. Lindsey and de Palma (2014) prove that the Mohring Harwitz cost recovery theorem holds also with uncertain demand. We confirm that the above intuition for a single road also applies to a public network. Tan and Yang (2012) and discuss flexible contracts for road franchising under demand uncertainty. Lu and Meng (2017a) and Lu and Meng (2017b) study build–operate–transfer (BOT) contracts for a single road and for one operator under demand uncertainty, where capacity must be set before the uncertain demand function is known. Just as in our paper, their analytical model allows uncertainty following any number of demand functions of any shape, while their numerical models allow only the intercept of the demand function to be uncertain, while its slope is certain. Feng, Zhang, Zhang, and Song (2018) only consider an uncertain demand intercept. To date, however, models with demand uncertainty have not been applied to

examine the private supply of roads in a mixed network.

This paper has two related aims. *First*, it investigates how uncertainty affects the private supply of roads in a network where there is also a public supply of roads. In reality, it is very common that private roads face competition from public roads, which often have a zero toll. *Second*, this paper studies how uncertainty changes the effect of regulation of private supply via a tender auction for the right to operate the road. Especially with uncertainty, direct regulation is difficult if only because the regulator will not know how much traffic there will be and what the price sensitivity will be that determines the possible market power mark-ups. The “generalized price auction” is won by the firm offering the lowest generalized price, which equals travel cost plus tolls paid. The “patronage auction” is won by the firm offering the highest usage of the road. Without uncertainty, these two auctions ensure the second-best zero-profit outcome, so that welfare is maximized under the constraint that the firm cannot make a loss. They thus fully remove any market power problems for welfare. With demand uncertainty, this is no longer true.

In other words, demand uncertainty makes regulating the private supply of roads more difficult and new forms of auctions need to be invented. The numerical simulation suggests that, with the help of competition, entry by regulation still works better than free entry. But contrary to the previous literature, the “generalized price auction” generates a higher expected welfare than the “patronage auction” for the parameter ranges considered. In addition, our study offers several new insights into the unregulated private supply of roads. For instance, when there exists a free complementary road and the road provider needs at least zero profit, contrary to the case without demand uncertainty, the toll is no longer equal to the marginal external congestion cost.

This paper is organized as follows. Section 2 discusses the analytical model for simple networks. Section 3 presents the simulation results for more complex networks. Section 4 concludes.

2. Analytical model

In this section, we study small networks with either serial links or parallel links. This means that we consider the purest types of link interactions (i.e. complementarity versus substitutability), and thus can identify the mechanisms that will occur in real life networks in the cleanest possible way. This provides us with economic intuition that carries over to larger and more realistic networks.

We aim to model demand uncertainty in a general way. To that end, let I denote the set of all possible demand states and i denote one particular state, so $i \in I$. Let p^i denote the probability of state i with $\sum_i p^i = 1$, N^i the total traffic flow in that state, and $D^i(N^i)$ the inverse demand in that state. By using $D^i(N^i)$, we can represent a variety of demand functions. Different demand states could have different functional forms and they could also have different parameters for the same form. After capacity is set and the demand state is realized, the firms are assumed to know the demand function of that state. This seems certainly realistic for long-term uncertainty, where firms have many years to do market research after starting to build their road. To keep things tractable, our model ignores the possibility of bankruptcy. It also ignores that uncertainty may raise the risk premium that firms or governments pay on loans used to finance new roads.

Our other assumptions are akin to those in standard models without uncertainty. There is a single market with one origin and one destination. Users are homogeneous, and risk neutral. For ease of presentation, we assume that all roads have the same travel cost function, which is denoted by $c(N_j^i, K_j)$ for state i and link j . We assume it increases with the flow, decreases with capacity, and is homogeneous of degree zero in

the ratio of flow to capacity. Mathematically, $\frac{\partial c(N_j^i, K_j)}{\partial N_j^i} > 0$, $\frac{\partial c(N_j^i, K_j)}{\partial K_j} < 0$ and $c(\theta N_j^i, \theta K_j) = c(N_j^i, K_j)$ for any θ . We assume that the marginal

¹ The reality is evidently more complex. Private firms may be restricted by several regulatory requirements or societal concerns. The public sector may care too much about revenue to the detriment of consumers or also care about pollution, fairness, or local economic development.

capacity cost, denoted by γ , is constant, to represent neutral scale economies in road construction. The toll for demand state i on link j is denoted by τ_j^i .

We discuss four regimes, which are natural benchmarks for evaluating the efficiency of alternative ways of organizing the private supply of roads. In the first-best case, the social planner maximizes the expected social welfare by setting the capacity and tolls of both links. This is the benchmark and the efficiency gain of any realistic policy can be evaluated against that of the first-best case. In the second-best case, the social planner faces a constraint that there is an unpriced link in the network, and optimizes over the capacity and toll of the other link. This will typically result in lower social welfare. In the second-best zero-profit case, the social planner faces an additional constraint: the provider earns a zero profit on the tolled link. This gives an upper limit for the achievable social welfare under private operations. Private firms will not operate under a loss, and competition will drive profits to zero. Finally, we study two competitive auctions as useful regulatory tools, which can generate the second-best zero-profit result in the deterministic case (Verhoef (2007)). In a patronage auction, the concession is awarded to the firm that will generate the highest expected level of use of the new road, while in a generalized price auction, the concession is granted to the firm that will offer the lowest expected generalized price, which is the sum of toll and time costs. We will discuss in the remainder of the section the equilibria of the four regimes for both serial and parallel networks.

2.1. Serial links

We first study a network of two serial links, where a traveler must use both links to get from the origin to the destination. These two serial links are thus perfect complements. The total traffic flow equals the traffic flow on each link, i.e. $N^i = N_0^i = N_1^i$. For the first-best and second-best cases, we present analytical expressions for the equilibrium tolls. For the other regimes, we discuss the properties of the tolls.

2.1.1. First-best for serial links

In the first-best case, the social planner maximizes the expected social welfare, which is the sum of the expected total consumer benefit minus the expected congestion cost and capacity cost. The choice variables are the capacities, state-dependent traffic flows, and state-dependent tolls. In addition, the user equilibrium constraints need to be satisfied: in any state the generalized price of an active route, i.e. the sum of user congestion cost and toll, equals the inverse demand in that state. So the social planner's problem can be expressed by the following Lagrangian.

$$\begin{aligned} \max_{K_0, N^i, \tau^i, \lambda^i} \sum_i p^i \cdot \left(\int_0^{N^i} D^i(n) dn - N^i \cdot (c(N^i, K_0) + c(N^i, K_1)) \right) - \gamma \cdot (K_0 + K_1) \\ + \sum_i \lambda^i \cdot (c(N^i, K_0) + c(N^i, K_1) + \tau^i - D^i(N^i)) \end{aligned} \quad (1)$$

For this and the following Lagrangians, we will skip the first-order conditions, which are taken w.r.t. the choice variables and the Lagrange multipliers. All variables, apart from the capacities and the multiplier for the zero-profit constraint (if relevant), are state-dependent. The solution is

$$\tau^i = N^i \cdot (c_{N^i}(N^i, K_0) + c_{N^i}(N^i, K_1)). \quad (2)$$

The optimal toll in each state i equals the marginal external congestion cost over the full trip in that state. It follows immediately that the two links are self-financing in expectation (Mohring and Harwitz (1962)). As De Vany and Saving (1977), Kraus (1982), D'Ouille and McDonald (1990), Arnott et al. (1996) and Lu and Meng (2017a) also show for related modeling settings, the optimal equilibrium capacity is larger than in the case without uncertainty. In intuitive words: both the

expected toll revenue and the total capacity cost are higher with uncertainty than without, but they are so by equal amounts. The intuition is that due to the convexity of the user cost function, the expected value of the marginal external cost over all states exceeds the marginal external cost for a deterministic traffic flow that is equal to the expected traffic flows under uncertainty. This raises the expected value of the toll, but also the optimal capacity of the road.

2.1.2. Second-best for serial links

To compare the private supply of a new road with the overall first-best case can be less informative when some untolled roads exist, which is very common in reality. In countries such as France it is even mandatory that there is an untolled alternative to a tolled motorway. A better benchmark would then be the case where the social planner can only set the capacity and tolls of a new road, but leaves the old road free of charge, as in the following Lagrangian.

$$\begin{aligned} \max_{K_1, N^i, \tau^i, \lambda^i} \sum_i p^i \cdot \left(\int_0^{N^i} D^i(n) dn - N^i \cdot (c(N^i, K_0) + c(N^i, K_1)) \right) - \gamma \cdot (K_0 + K_1) \\ + \sum_i \lambda^i \cdot (c(N^i, K_0) + c(N^i, K_1) + \tau^i - D^i(N^i)) \end{aligned} \quad (3)$$

The resulting expected welfare is generally lower than in the overall first-best case, as the capacity of the free road is suboptimal. Nevertheless, it is simple to show that the resulting toll in each demand state equals the marginal external congestion cost of the full trip:

$$\tau^i = N^i \cdot (c_{N^i}(N^i, K_0) + c_{N^i}(N^i, K_1)). \quad (4)$$

The expected toll revenue more than compensate for the capacity cost of the toll link. In fact, the revenue would cover the cost of supplying both links at the optimal capacity, because with an unpriced perfectly complementary link, the second-best toll becomes equal to what would have been the first-best tolls for the two links together.

2.1.3. Second-best zero-profit for serial links

Compared to the second-best case, the second-best zero-profit case is probably a better benchmark for competing private firms, as with free entry and competitive auctions, profit will be driven down to zero. It is also a good benchmark for regulation, as it identifies the most efficient outcome under the constraint that toll revenue covers capacity cost. In addition to the user equilibrium constraints, we need to add a zero-profit constraint.

Proposition 1. (Second-best zero-profit for serial links).

Suppose there are two serial links of which only link 1 is priced and this link must have a zero expected profit. The second-best zero-profit toll differs from the marginal external cost of $MEC_1^i = N^i \cdot (c_{N^i}(N^i, K_1))$ as long as

$$\frac{c_{N^i}(N^i, K_0)}{D_{N^i}^i(\cdot) - c_{N^i}(N^i, K_0)} \text{ is not constant across the equilibria of all states.}$$

Remark. The $\frac{c_{N^i}(N^i, K_0)}{D_{N^i}^i(\cdot) - c_{N^i}(N^i, K_0)}$ is unlikely to be constant, as it contains the derivatives of cost and demand. It can only be constant in two exceptional cases: i) when demand and costs are both linear and the uncertainty only affects the demand intercept, and ii) when demand always has an infinite price sensitivity (i.e. $D_{N^i}^i(\cdot) = 0$ in all states).

Proof. In the second-best zero-profit case, a social planner maximizes social welfare under the constraint that the profit on link 1 is zero. The corresponding Lagrangian is the following.

$$\begin{aligned} \max_{K_1, N^i, \tau^i, \lambda^i, \lambda^{zp}} \sum_i p^i \cdot \left(\int_0^{N^i} D^i(n) dn - N^i \cdot (c(N^i, K_0) + c(N^i, K_1)) \right) - \gamma \cdot (K_0 + K_1) \\ + \sum_i \lambda^i \cdot (c(N^i, K_0) + c(N^i, K_1) + \tau^i - D^i(N^i)) + \lambda^{zp} \cdot (\sum_i p^i \cdot N^i \cdot \tau^i - \gamma \cdot K_1) \end{aligned} \quad (5)$$

Here, λ^i for the user equilibrium constraint is state-dependent, but

λ^p for the zero-profit constraint is not. After simplification, we show for any i the following condition should be satisfied.²:

$$\lambda^p = \frac{\tau^i - N^i(c_{N^i}(N^i, K_0) + c_{N^i}(N^i, K_1))}{-\tau^i - N^i(D_{N^i}^i(\cdot) - c_{N^i}(N^i, K_0) - c_{N^i}(N^i, K_1))} \quad (6)$$

If the toll were to equal the marginal external cost in every state, i.e. $\tau^i = N^i c_{N^i}(N^i, K_1)$, the condition could be further simplified to $\lambda^p = \frac{c_{N^i}(N^i, K_0)}{D_{N^i}^i(\cdot) - c_{N^i}(N^i, K_0)}$. Only when this fraction is constant across all states is there a solution for λ^p . If this implausible condition is not met, the equilibrium toll must differ from the marginal external cost.

The result in Proposition 1 differs critically from that under certain demand in Verhoef (2008), where the second-best zero-profit outcome results in a toll equal to the marginal external cost on the priced link. With demand uncertainty, the toll in any demand state generally differs from the marginal external cost on link 1 in that state, unless demand and cost are always exactly linear and demand uncertainty never affects the slope of the demand function. This is unlikely to be true in reality. Indeed, engineering studies typically find highly convex functions, such as the famous BPR function, which has a flow to the fourth power (e.g. Small and Verhoef (2007)).

The intuition behind Proposition 1 is that in second-best optimum, a shift of one dollar of revenue between two states should bring as much benefit in the one state as damage in the other. As a result, contrary to the deterministic case, the toll cannot equal the marginal external cost. For example, for a linear inverse demand function $D^i(N^i) = d_0^i - d_1 \cdot N^i$ and two demand states, such as $d_0^h > d_0^l$, the Pigouvian toll directly implies $\frac{-c_{N^h}(N^h, K_0)}{c_{N^h}(N^h, K_0) + d_1} < \frac{-c_{N^l}(N^l, K_0)}{c_{N^l}(N^l, K_0) + d_1}$ when $N^h > N^l$, and thus no solution for λ^p . In other words, the social welfare is not maximized with the Pigouvian toll, because it can be increased further by decreasing N^h and increasing N^l .

2.1.4. Auctions for serial links

So far, we have discussed only the public supply of roads under various constraints. However, sometimes the private supply of roads may be preferable to the public supply: for instance, due to the lack of public funds, higher efficiency in the private sector, or just because roads are not a public good. There is however the problem that private roads will tend to have market power. This could be solved by direct regulation, but this might be difficult due to a lack of information for the regulator. This is especially relevant if demand or costs are uncertain. Then a competitive auction on expected use (patronage) or expected generalized price is interesting, as then the competition in the auction drives the outcome towards the second-best zero-profit outcome. Nevertheless, as we will see, auctions do not lead to exactly the same outcome as in the second-best zero-profit case, differing in this regard from the deterministic setting.

We assume a perfectly competitive auction, all bidding firms have the same marginal capacity cost γ and full information of the congestion cost function, and they will bid until the expected profit is exhausted. In the deterministic case, Verhoef (2008) showed that two auctions can implement the second-best zero-profit outcome. These are the patronage auction, where firms bid in terms of committing to achieve the highest traffic flow on the toll road and the generalized price auction, where they bid to realize the lowest generalized price. So we will now study how these two auctions perform under uncertain demand. In reality, auctions will tend to have imperfect competition: the number of

bidders is limited as only a few firms (operating in a country or regions) have the expertise to build and operate toll roads, and firms may differ in construction and management costs or have different expectations. Van den Berg and Rouwendal (2016) find that under deterministic demand, the outcome of an auction approaches the perfectly competitive case as the auction becomes more competitive (as the bidding firms become more similar).

Proposition 2. (Patronage auction for serial links).

If road 0 remains unpriced, the equilibrium toll on serial road 1 of a perfectly competitive patronage auction differs from:

- I the equilibrium toll of the second-best zero-profit case, under any structure of the demand uncertainty.
- II the marginal external congestion cost MEC_1^i of road 1, unless $\frac{-1}{N^i(D_{N^i}^i(\cdot) - c_{N^i}(N^i, K_0))}$ is constant across the equilibria of all states.

Proof. To maximize expected patronage, under the user equilibrium constraint in every state and the zero expected-profit constraint, the Lagrangian is

$$\max_{K_1, N^1, \tau^1, \lambda^1, \lambda^p} \sum_i p^i \cdot N^i + \sum_i \lambda^i \cdot (c(N^i, K_0) + c(N^i, K_1) + \tau^i - D^i(N^i)) + \lambda^p \cdot (\sum_i p^i \cdot N^i \cdot \tau^i - \gamma \cdot K_1). \quad (7)$$

We can again use the Lagrange multiplier for the zero-profit constraint:³

$$\lambda^p = \frac{1}{-\tau^i - N^i(D_{N^i}^i(\cdot) - c_{N^i}(N^i, K_0) - c_{N^i}(N^i, K_1))} \quad (8)$$

To show result 2.I., that the tolls from the patronage auction differ from the second-best zero-profit outcome, we only need to compare the multipliers in (6) and (8). If the two policies were to lead to the same toll in each state, the numerators would need to be proportional, since the denominators are the same. In Eq. (6), the numerator is clearly not constant over states and so cannot be proportional to the numerator in (8), which equals 1.

To prove 2.II., that the toll generally differs from $MEC_1^i = N^i \cdot c_{N^i}(N^i, K_1)$, we again use the fact that λ^p must be constant across states. If the toll were to equal MEC_1^i , then λ^p would simplify to $\frac{-1}{N^i(D_{N^i}^i(\cdot) - c_{N^i}(N^i, K_0))}$, which has to be the same in any state for the multiplier to be constant across states.

It is very unlikely for the above condition to be met in reality. For instance, assume that the functional form of the demand is constant across states; then, for the condition to hold, the demand in all states must be $D^i = d_0^i + c(N^i, K_0) - d_1 \cdot \log(N^i)$. For this demand function, the uncertainty can only be in the demand intercept d_0^i , and the demand function happens to contain the cost function of link 0 at arbitrary capacity K_0 as well as the term $d_1 \cdot \log(N^i)$. Besides the very arbitrary and exact nature of this demand set-up, this also means that the demand is upward sloping in some range unless the cost is linear.

Proposition 3. (Generalized price auction for serial links)

If road 0 remains unpriced, the equilibrium toll on serial road 1 of a perfectly competitive generalized price auction differs from:

- I the equilibrium toll of the second-best zero-profit case, for any demand uncertainty.
- II the equilibrium toll of the patronage auction, unless the slope of the

² λ^p reflects how much the expected social welfare changes if we allow a small expected surplus or deficit. The numerator is the derivative of the social welfare in state i with respect to the traffic volume in state i . It thus equals the height of the Harberger triangle that measures the deadweight loss due to inefficient tolling. The denominator is the derivative of the financial deficit in state i with respect to the traffic volume in state i . As the traffic volume in state i increases, the social planner gets more toll payments from the new marginal traveler, but collects less toll from every original traveler.

³ This multiplier reflects how much the expected patronage changes if we allow a small expected deficit. It has to be the same across states. The numerator equals the derivative of the traffic volume in a state (directly entering the objective as the patronage) with respect to itself, and is therefore 1. The denominator is the derivative of the expected deficit in state i with respect to the traffic volume in that state, which is the same as in (6).

inverse demand is constant across the equilibria of all states.⁴
 III the marginal external cost MEC_1^i of road 1, unless $\frac{D_{N^i}^i(\cdot)}{(N^i(D_{N^i}^i(\cdot) - c_{N^i}(N^i, K_0)))}$
 is constant across states.

Proof. To maximize the expected generalized price, under the user equilibrium constraint in every state and the zero expected-profit constraint, the Lagrangian is

$$\min_{K_1, N_1^i, \tau^i, \lambda^i, \lambda^p} \sum_i p^i \cdot D^i(N^i) + \sum_i \lambda^i (c(N^i, K_0) + c(N^i, K_1) + \tau^i - D^i(N^i)) + \lambda^p (\sum_i p^i \cdot N^i \cdot \tau^i - \gamma \cdot K_1). \quad (9)$$

The multiplier for the zero expected-profit constraint is

$$\lambda^p = \frac{D_{N^i}^i(\cdot)}{-\tau^i - N^i \cdot (D_{N^i}^i(\cdot) - c_{N^i}(N^i, K_0) - c_{N^i}(N^i, K_1))}. \quad (10)$$

Using the methods used for Proposition 2, it is straightforward to show that the tolls resulting from the price auction generally differ from those of 3.I, the second-best zero-profit case, and from 3.III, the marginal external cost. So we omit a detailed Proof. For result 3.II, we start with a linear inverse demand function with uncertainty only on the intercept, where $D_{N^i}^i(\cdot)$ is a constant and the same in all states. Then the zero-profit multipliers (7) and (9) for the two auctions are proportional. Therefore, they can have the same toll rules and both have zero-profit multipliers that are constant over demand states. The intuition is that, in this situation, the maximum expected total traffic flow corresponds to the minimum expected generalized price. For non-linear demands or more general uncertainty, the outcomes of the auctions will be different, as there is no one-to-one relation between expected flows and expected generalized price, meaning that both zero-profit multipliers cannot be constant for the same toll rule.

In sum, the patronage auction and the generalized price auction in general cannot replicate the result for the second-best zero-profit case with serial links if there is demand uncertainty, and the two auctions also generally lead to different outcomes. This differs from certain demand, where both auctions always lead to the second-best zero-profit outcome. There are two ways to explain this. Firstly, the auctions and the second best zero-profit case have different *expected* optimization objectives. For linear inverse demand functions, in each state, the consumer surplus is quadratic in the traffic flow while the patronage and the generalized price are linear in the traffic flow, so the highest *expected* consumer surplus corresponds to neither the highest *expected* patronage nor the lowest *expected* generalized price. Secondly, although the FOCs for τ^i , K_1 , λ^i and λ^p are the same for the two auctions and the second-best zero-profit case, they cannot determine a unique solution. The solution depends also on the FOCs for N^i , which differ for different cases. On the contrary, when there is no demand uncertainty, the three maximizations are equivalent and the FOCs determine a unique combination of flows, capacity, and toll (Wu, Yin, and Yang (2011) and Verhoef (2007)).

2.2. Parallel links

This section considers two parallel links, where both roads connect the same origin and destination and a traveler can use either of them. They are pure substitutes, so the total traffic flow is the sum of the traffic flow of both links, i.e. $N^i = N_0^i + N_1^i$.

2.2.1. First-best for parallel links

In the first-best case, the Lagrangian is as follows.

⁴ So, in practice, this means that demand must be exactly linear and that uncertainty can not affect the demand slope.

$$\max_{K_j, N_j^i, \tau_j^i, \lambda_j^i} \sum_i p^i \cdot \left(\int_0^{N_0^i + N_1^i} D^i(n) dn - N_0^i \cdot c(N_0^i, K_0) - N_1^i \cdot c(N_1^i, K_1) \right) - \gamma \cdot (K_0 + K_1) + \sum_j \sum_i \lambda_j^i \cdot (c(N_j^i, K_j) + \tau_j^i - D^i(N_0^i + N_1^i)) \quad (11)$$

The result is

$$\tau_j^i = N_j^i \cdot c_{N_j^i}(N_j^i, K_j). \quad (12)$$

The Pigouvian toll is levied on each link in each state, so the externality is internalized and the roads are self-financing in expectation (De Vany and Saving (1977); Kraus (1982); D'Ouille and McDonald (1990); Arnott et al. (1996); and Lu and Meng (2017a)).

2.2.2. Second-best for parallel links

In the second-best case, now an untolled parallel link already exists. When the social planner optimizes the capacity and tolls of the new road, the Lagrangian is

$$\max_{K_1, N_1^i, \tau_1^i, \lambda_1^i} \sum_i p^i \cdot \left(\int_0^{N_0^i + N_1^i} D^i(n) dn - N_0^i \cdot c(N_0^i, K_0) - N_1^i \cdot c(N_1^i, K_1) \right) - \gamma \cdot (K_0 + K_1) + \sum_i \lambda_0^i \cdot (c(N_0^i, K_0) - D^i(N_0^i + N_1^i)) + \sum_i \lambda_1^i \cdot (c(N_1^i, K_1) + \tau_1^i - D^i(N_0^i + N_1^i)). \quad (13)$$

The solution for the toll is

$$\tau_1^i = N_1^i \cdot c_{N_1^i}(N_1^i, K_1) + N_0^i \cdot c_{N_0^i}(N_0^i, K_0) \cdot \frac{D_{N_1^i}^i(N_0^i + N_1^i)}{c_{N_0^i}(N_0^i, K_0) - D_{N_0^i}^i(N_0^i + N_1^i)}. \quad (14)$$

Similar to the case without uncertainty, the toll in each demand state equals the sum of the marginal external congestion cost and a negative network spillover effect. The social planner should expect a loss on the toll road, as the toll is lower than the Pigouvian toll (Verhoef (2007)).

2.2.3. Second-best zero-profit for parallel links

In the second-best case with parallel links, the operator of the toll road makes a loss. The logical follow-up question is: what is the highest expected social welfare if this road has to be self-financing? This is a natural benchmark for private supply when there is an untolled alternative parallel road: we cannot expect the private road to make a loss, so the best we can hope for is welfare maximization under a zero-profit constraint.

Proposition 4. (Second-best zero-profit for parallel links).

In contrast to the case with the serial links, with parallel links, the second-best zero expected-profit toll can be Pigouvian, as it equals the marginal external cost MEC_1^i on link 1 in each demand state i . A necessary condition is that the cost function be such that for any capacities K_j , the marginal social cost at any flow N_j equals the travel cost for a flow that is a factor θ higher: $\frac{\partial(N_j \cdot c(N_j, K_j))}{\partial N_j} = c(\theta \cdot N_j, K_k)$.

Remark The condition in Proposition 4 holds for most commonly used cost functions, such as the BPR function of $c_0 + c_1 \cdot \left(\frac{N_j^i}{K_j}\right)^\alpha$, the

Vickrey bottleneck congestion, and an exponential cost of $c_0 + c_1 \cdot e^{\frac{N_j^i}{K_j}}$. It would not hold for a cost function that has more variability in the congestion than commonly used in the literature, such as $c_0 + c_1 \cdot \left(\frac{N_j^i}{K_j}\right) + c_2 \cdot \left(\frac{N_j^i}{K_j}\right)^2$.

Proof. Mathematically, we solve the following Lagrangian problem.

$$\begin{aligned}
& \max_{K_1, N_1^i, \tau_1^i, \lambda_j^i, \lambda^{\mathcal{P}}} \sum_i p^i \cdot \left(\int_0^{N_0^i + N_1^i} D^i(n) dn - N_0^i \cdot c(N_0^i, K_0) - N_1^i \cdot c(N_1^i, K_1) \right) \\
& - \gamma \cdot (K_0 + K_1) \\
& + \sum_i \lambda_0^i \cdot (c(N_0^i, K_0) - D^i(N_0^i + N_1^i)) \\
& + \sum_i \lambda_1^i \cdot (c(N_1^i, K_1) + \tau_1^i - D^i(N_0^i + N_1^i)) \\
& + \lambda^{\mathcal{P}} \cdot (\sum_i p^i \cdot \tau_1^i \cdot N_1^i - \gamma \cdot K_1)
\end{aligned} \quad (15)$$

After simplification, the condition for the Lagrange multiplier is ⁵

$$\lambda^{\mathcal{P}} = \frac{\tau_1^i - N_1^i \cdot c_{N_1^i}(\cdot) - N_0^i \cdot c_{N_0^i}(\cdot) \frac{D_{N_1^i}^i(\cdot)}{c_{N_0^i}(\cdot) - D_{N_0^i}^i(\cdot)}}{-\tau_1^i - N_1^i \cdot \left(D_{N_1^i}^i(\cdot) + D_{N_0^i}^i(\cdot) \frac{D_{N_1^i}^i(\cdot)}{c_{N_0^i}(\cdot) - D_{N_0^i}^i(\cdot)} - c_{N_1^i}(\cdot) \right)} \quad (16)$$

With the Pigouvian toll, i.e. $\tau^i = N_1^i \cdot c_{N_1^i}(N_1^i, K_1)$, the multiplier further simplifies to $\lambda^{\mathcal{P}} = \frac{N_0^i}{N_1^i}$. Moreover, in each state, the generalized price on link 1 will equal the marginal social cost of $MSC_1^i = \frac{\partial(N_1^i \cdot c(N_1^i, K_1))}{\partial N_1^i}$ and on link 0 the price will be simply $c(N_0^i, K_0)$. Given our assumptions on the congestion cost function and the condition in the proposition, $\frac{N_0^i}{N_1^i}$ is constant across states.⁶

If the social planner allows an expected deficit on the tolled road, the positivity of $\lambda^{\mathcal{P}}$ in (15) shows that the expected welfare increases. The Pigouvian toll on the toll road does not take into account the congestion spill-over to the untolled road, and hence lowering the toll below the Pigouvian level would raise welfare. This increase in welfare is larger when there are more travelers on the untolled road or fewer travelers on the toll road. In the extreme case of zero traffic flow on the untolled road, the effect on welfare is zero because we have already achieved the highest possible welfare.

2.2.4. Auctions for parallel links

Without demand uncertainty, the patronage auction and the generalized price auction would replicate the second-best zero-profit outcome also for parallel links (Verhoef, 2007). This subsection shows that this is not true with demand uncertainty. For policy makers thinking of how to regulate the private firms, more sophisticated mechanisms are needed.

Proposition 5. (Patronage auction for parallel links).

Unlike the case without demand uncertainty, when there is uncertain demand, the perfectly competitive patronage auction generally cannot attain the second-best zero-profit auction, and hence leads to a lower welfare. A sufficient condition for this is the condition in Proposition 4.

Proof. For the patronage auction, we set up the Lagrangian that maximizes the expected flow on the tolled link subject to user equilibrium constraints and a zero-profit constraint:

⁵ The numerator of $\lambda^{\mathcal{P}}$ is the derivative of welfare in state i w.r.t. N_1^i . It equals the sum of the heights of the Harberger triangles of both links, where that of the untolled link is weighted to reflect the substitution in usage of the two links. The denominator is the derivative of the deficit in state i w.r.t. N_1^i . As the tolled flow increases, the road provider gets toll payments from new tolled users, but can get a lower toll from all original tolled users, as the general prices on the two links must be the same. If N_1^i marginally increases

by Δ , N_0^i must decrease by $\frac{D_{N_1^i}^i(\cdot)}{c_{N_0^i}(\cdot) - D_{N_0^i}^i(\cdot)} \Delta$.

⁶ For instance, under BPR costs of $c_0 + c_1 \left(\frac{N_1^i}{K_1} \right)^\chi$, the resulting flows will satisfy

$\frac{N_0^i}{N_1^i} = (1 + \chi)^{1/\chi} \frac{K_0}{K_1}$ under a Pigouvian toll on link 1 in any state i .

$$\begin{aligned}
& \max_{K_1, N_1^i, \tau_1^i, \lambda_j^i, \lambda^{\mathcal{P}}} \sum_i p^i \cdot N_1^i + \sum_i \lambda_0^i \cdot (c(N_0^i, K_0) - D^i(N_0^i + N_1^i)) \\
& + \sum_i \lambda_1^i \cdot (c(N_1^i, K_1) + \tau_1^i - D^i(N_0^i + N_1^i)) + \lambda^{\mathcal{P}} \cdot (\sum_i p^i \cdot \tau_1^i \cdot N_1^i - \gamma \cdot K_1).
\end{aligned} \quad (17)$$

The FOCs for τ_1^i , K_1 , λ_j^i and $\lambda^{\mathcal{P}}$ are the same as those of the second-best zero-profit case. However, unlike the case without uncertainty Wu et al. (2011), these conditions cannot determine a unique solution, since demand uncertainty brings in more choice variables. The Lagrange multiplier for the zero-profit constraint shows that the toll, which has no closed-form solution, cannot be the same as in the second-best zero-profit case⁷:

$$\lambda^{\mathcal{P}} = \frac{1}{-\tau_1^i - N_1^i \cdot \left(D_{N_1^i}^i(\cdot) + D_{N_0^i}^i(\cdot) \frac{D_{N_1^i}^i(\cdot)}{c_{N_0^i}(\cdot) - D_{N_0^i}^i(\cdot)} - c_{N_1^i}(\cdot) \right)} \quad (18)$$

If the toll were to equal the marginal external cost in each state i , then $\lambda^{\mathcal{P}}$ could not be constant, and so this toll rule does not maximize the Lagrangian that is equivalent to the patronage auction. So, unlike the case without demand uncertainty, the resulting toll cannot be equal to the marginal external congestion cost on the toll road.

As this auction leads to a zero profit but a different outcome than the second-best zero-profit case, it must have a lower welfare, since the second-best zero-profit case by definition attains the highest welfare given that road 0 is unpriced and road 1 has to have zero profit.

Now we turn to the perfectly competitive generalized price auction. It implies that a firm minimizes the expected generalized price under the limitation that the tolled road breaks even in expected terms, since the perfect competition in the auction implies that the auction winner must attain a zero expected pay-off.

Proposition 6. (Generalized price auction for parallel links).

The perfectly competitive generalized price auction with an untolled parallel alternative leads to a different equilibrium toll than:

- I the marginal external cost, unless the flow N^i is the same in all demand states.
- II the toll in the second-best zero-profit case. A sufficient condition for this is the condition in Proposition 4.
- III the toll in the patronage auction, unless demand and cost functions are both linear in traffic flow and uncertainty is only in the intercept of the demand function.

Proof. The Lagrangian for the problem with parallel links is

$$\begin{aligned}
& \min_{K_1, N_1^i, \tau_1^i, \lambda_j^i, \lambda^{\mathcal{P}}} \sum_i p^i \cdot D^i(N_0^i + N_1^i) + \sum_i \lambda_0^i \cdot (c(N_0^i, K_0) - D^i(N_0^i + N_1^i)) \\
& + \sum_i \lambda_1^i \cdot (c(N_1^i, K_1) + \tau_1^i - D^i(N_0^i + N_1^i)) + \lambda^{\mathcal{P}} \cdot (\sum_i p^i \cdot \tau_1^i \cdot N_1^i - \gamma \cdot K_1).
\end{aligned} \quad (19)$$

After some substitutions, $\lambda^{\mathcal{P}}$ can be expressed as follows.⁸

$$\lambda^{\mathcal{P}} = \frac{D_{N_1^i}^i(\cdot) + D_{N_0^i}^i(\cdot) \frac{D_{N_1^i}^i(\cdot)}{c_{N_0^i}(\cdot) - D_{N_0^i}^i(\cdot)}}{-\tau_1^i - N_1^i \cdot \left(D_{N_1^i}^i(\cdot) + D_{N_0^i}^i(\cdot) \frac{D_{N_1^i}^i(\cdot)}{c_{N_0^i}(\cdot) - D_{N_0^i}^i(\cdot)} - c_{N_1^i}(\cdot) \right)} \quad (20)$$

If the toll were to equal the MEC^i in each state, the $\lambda^{\mathcal{P}}$ in (21) would simplify to $1/N^i$ and this varies if the flow is not the same in all states. This proves result 6.I. A road use that does not vary with demand

⁷ The numerator of $\lambda^{\mathcal{P}}$ is the derivative of the traffic volume on the tolled road in state i with respect to itself, thus its value is 1. The denominator is the same as in (15).

⁸ The numerator is the derivative of the generalized price in state i with respect to the tolled flow, taking into consideration the induced traffic volume on the untolled link. The denominator is the same as in the earlier two cases, (17) and (19).

Table 1
Benchmark equilibria.

Variables	Base	First-best	Second-best	Second-best zero-profit
$E[S]$	60189	113741	112193	103241
ω	0	1	0.971	0.804
$E[\pi_{a0}], E[\pi_{b0}]$	–5250	0	–5250	–5250
$E[\pi_{a1}], E[\pi_{b1}]$	–	–	–8043	0
K_{a0}, K_{b0}	1500	3644	1500	1500
K_{a1}, K_{b1}	–	–	2479	1401
t_{a0}^h, t_{b0}^h	0	4.226	0	0
t_{a1}^h, t_{b1}^h	–	–	0.350	4.226
t_{a0}^l, t_{b0}^l	0	1.119	0	0
t_{a1}^l, t_{b1}^l	–	–	0.022	1.119
c_{a0}^h, c_{b0}^h	14.230	2.931	3.313	7.157
c_{a1}^h, c_{b1}^h	–	–	2.963	2.931
c_{a0}^l, c_{b0}^l	6.697	2.155	2.131	3.273
c_{a1}^l, c_{b1}^l	–	–	2.110	2.155
p^h	28.459	14.315	6.626	14.315
p^l	13.395	6.546	4.263	6.546

uncertainty seems very implausible. Under the sufficient condition for result 6.II, it would be impossible that (21) were constant at the second-best zero-profit case that that we know makes (17) constant across states. For 6.III, the two auction outcomes differ, since the multipliers in (19) and (21) differ in their numerators, while the denominators are the same. For the numerator in (21) to be constant over all states, as in (18), a sufficient condition is that the demand and cost functions are both linear in the traffic flow and the uncertainty is only in the intercept of the demand function.

In sum, neither the patronage auction nor the generalized price auction can in general replicate the result of the second-best zero-profit case. The traffic volumes that maximize the expected social welfare under the zero-profit constraint neither maximize the expected patronage on the toll road nor minimize the expected generalized price.

3. Numerical analysis

The analytical results for the two simple networks show that demand uncertainty is likely to affect the efficiency of the private supply of roads, in that equilibria with competitive auctions differ from the second-best zero-profit outcomes. For a more general network, which is also more realistic, clear-cut analytical results are hard to obtain, and we rely on numerical simulations to gain further insights. Moreover, with the help of simulations, not only may we examine statically how private roads perform in a given mixed network, but can also learn how to regulate dynamically the private provision of roads through the formation of networks.

Similar to Verhoef (2008), we assume there are two serial segments a and b in a network connecting one origin and one destination. The initial links on the two segments, denoted by $a0$ and $b0$, are untolled, which represent a pre-existing free public road network. Private firms can add capacity to each segment, one at a time, and then charge tolls. For example, if the first firm adds a link in section a , we denote the new link by $a1$. Road $a1$ is now parallel to the existing link $a0$, and serial to the existing link $b0$. In this way, we can model both parallel and serial competition and the development of a network.

The timing of the game is as follows: since construction takes time, there is, at the beginning of each round, uncertainty about the future demand. Firms compete to add capacity to one section of the network, without knowing the demand that will be realized. After the demand uncertainty is resolved, the firms can no longer change their capacities, but they can decide on the tolls to charge on their own links. Then there is again demand uncertainty, and a new round begins. This sequential game continues until there is no profit for a new entry.

We will discuss the basic benchmarks first, then compare and contrast two regimes: the unregulated free-entry regime and the regulated entry-by-auction regime. In the free-entry regime, the firm with the highest expected profit adds capacity to the most desired section. After the capacity is built, the demand is known, and all firms in the network play a Bertrand price setting game, i.e. every firm sets its own toll simultaneously while taking the tolls of the other road operators as given. In the entry-by-auction regime, the winner of an auction can add capacity. Due to the perfect competition in the auctions, any firm that adds capacity earns zero profit in expectation. The auction can be on either the expected patronage of the new road, or the expected generalized price. To be comparable to the deterministic case in Verhoef (2008), when demand is known, all existing firms charge tolls as promised in the auction and stick to these over successive rounds, so there is no direct toll competition in the entry-by-auction regime.

The parameters of the numerical simulation are as follows.⁹ To be comparable with Verhoef (2008), the inverse demand function is linear and $D^i(N^i) = \delta_0^i - \delta_1 \cdot N^i$. The demand uncertainty is in the intercept, which is δ_0^h with probability p^h and δ_0^l with probability $1 - p^h$. We set $\delta_1 = 0.01167$, $\delta_0^h = 74.11$, $\delta_0^l = 49.41$, $p^h = 0.5$, which means that compared to the expected level, the reservation price can go up or down by 20% with equal probability. According to Flyvbjerg et al. (2006), actual traffic deviates from the forecasted traffic by more than 20% for half of the road projects. The congestion cost function is of the BPR form $c(N_j^i, K_j) = \alpha \cdot t_f \cdot \left(1 + \beta \cdot \left(\frac{N_j^i}{K_j}\right)^\chi\right)$. The value of time, α , is set at 7.5. t_f is the free-flow travel time and is set at 0.25, implying a total trip length of 60 km for a highway with a speed of 120 km/hour. β and χ take their conventional values of 0.15 and 4, respectively. The marginal capacity cost is 3.5 for both segments, and represents the hourly capital cost per unit of capacity. We assume initial capacities of .

3.1. Benchmark

As a benchmark, Table 1 summarizes the characteristics of the base, first-best, second-best, and second-best zero-profit cases. The results of interest are: the social welfare (S), relative efficiency (ω)¹⁰, profit (π),

⁹ The assumption of a linear demand function and two-state demand uncertainty is for simplicity of simulation and the results does not change qualitatively when varying these assumptions. We have discussed general forms of demand function and uncertainty distribution in the theoretical section.

¹⁰ The relative efficiency is the gain in the expected social welfare in the regime, divided by the gain achieved when moving from the base equilibrium to the first-best outcome.

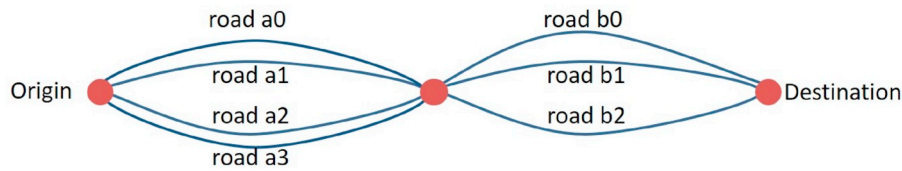


Fig. 1. The network after 5 rounds of entry.

Note: The origin and destination are connected by the segments *a* and *b*, where each has a number of parallel roads. Initial capacities *a*₀ and *b*₀ will remain unpriced. We assume that entry starts on segment *a*. During each following entry, it is more profitable to enter on the segment with the lower total capacity.

capacity (K), toll (τ), congestion cost (c), and generalized price (P). The superscript h denotes the high demand state, and l the low one. The subscript a (b) is for section a (b), while 0 (1) is for the initial (newly-added) link. E is for expectation. The results are fully consistent with the analytical model.

The base equilibrium with the two untolled links is quite congested, and its expected social welfare is only half the first-best value. Since no toll is charged, there is a loss on the two initial links. In the first-best case, the capacity more than doubles from the base case and the congestion cost in both states decreases. The expected profit is zero, because the profit in the high state and the loss in the low state cancel out. Compared with the deterministic case discussed in Verhoef (2008), the first-best capacity is larger under uncertainty, as was also found in Lu and Meng (2017a).

The second-best case achieves 97.1% of the increase in the expected welfare from the base equilibrium to the first-best case. However, this generates a considerable loss for the two newly-added parallel links in both demand states, because the capacity expansion is too large for the toll revenue to cover. As predicted by our theory for the parallel links, the second-best zero-profit case has the same toll and generalized price as in the first-best case in both demand states. It can achieve 80.4% of the increase in social welfare.

3.2. Entry games

We will look at three entry regimes: free entry, entry by patronage auction, and entry by generalized price auction. On both segments a and b there will be some initial public capacity of 1500, so that we start in round 0 with the base case. Thereafter, in each round, firms first compete to add capacity to one section of the network, without knowing the demand function that will be realized. After the demand function becomes known, all firms decide on the tolls to charge. Then a new round begins and there is again demand uncertainty. In this way, we study the evolution of the addition of capacity by firms over time, which mimics the slow and adaptive process of the expansion of road capacity. Firms will always choose to enter on the segment with the lower capacity, as this is more profitable. Fig. 1 shows how the network looks after 5 rounds of entry.

We will compare the entry regimes with the second-best zero-profit case and with the base case, in terms of capacity, generalized price, and welfare. Each entry game starts at the base case, and hence does at least as well. The best we can hope for in terms of welfare from a competitive case is the second-best zero-profit outcome, as entering firms cannot make an expected loss and there will remain some unpriced public capacity.

For simplicity, we assume that within each round, firms are forward looking and rational, so the capacity decision takes into account the equilibrium toll setting in the next stage. But between rounds, firms are assumed myopic, in the sense that they take every round as being the last, until they are “surprised” by newcomers who change the network structure. We make these assumptions to avoid unsolvable dynamic games, and we think they may also represent the slow and lump-sum development in infrastructure in real life, because it usually takes considerable time for new roads to materialize, a firm can focus on competing with the existing firms for now and not worry too much about possible new competitors in the future. But we admit this is a

simplification. In general, as also found by Van den Berg and Verhoef (2012), if firms can anticipate new entries, they will set larger capacities to limit or preempt the entry.

Another assumption is that when the uncertainty is assumed to be resolved after completing an investment, there is again uncertain demand in a new round. The replication of the same type of uncertainty when every round begins is of course unrealistic, but helpful in our setting, in that it avoids making ad hoc assumptions on how the uncertainty itself evolves over time. In addition, this setting would truthfully represent the case of peak-load pricing, when the demand fluctuates regularly within a period in which capacity is fixed.

3.2.1. Free entry

In the free entry regime, the firm with the highest expected profit sets some capacity on the segment of its choice. Then the demand uncertainty is resolved, and firms set their tolls simultaneously. We allow both old and new firms to add capacity. But, just as in Verhoef (2008), if a firm already has some capacity on one segment but not on the other, it will always have a higher gain from entering than existing firms on that segment or firms that have no capacity at all. Therefore, our assumption means that this firm will enter. So there are no problems with double marginalization, in the sense that one may expect firms to be active on both serial segments, and competition between such firms on parallel segments to drive down tolls.

Fig. 2 shows the evolution of the capacities round by round. We start in round 0 with the initial base case capacities. We may assume, without loss of generality, that capacity is first added on section a . In equilibrium, the firm that has added capacity on section a will, in the next round, add capacity on section b . Then a new firm adds capacity on section a , then this same firm will add on section b , and so on. This pattern emerges because (1) the same firm can better coordinate the tolls on both sections, so it is the same firm rather than a new one which will invest on a serial segment in the even round; (2) when the capacity on one section is expanded, it is more profitable to add capacity on the complementary section. In addition, the capacity addition in section b is always larger than that in section a in the previous round, because of the increased demand due to the capacity expansion on the other link. A new firm always adds capacity in section a in the next round because if an old firm does so, it will end up competing with its own capacities in section a and gets a smaller profit gain than a new firm.

Capacity steadily increases with entry. But even after eight rounds, the total capacity is well below that in the second-best zero-profit. As Figs. 3 and 4 show, the welfare is also still well below that in the second-best zero-profit case, and the generalized price is much higher. Eight rounds of entry, and so four competing firms, probably is more than we can expect in reality, so that clearly free entry leads to a welfare loss and hurts consumers. The qualitative patterns match those described in Verhoef (2008) and Van den Berg and Verhoef (2012) for deterministic demand.

3.2.2. Entry by perfectly-competitive auctions

Now we turn to the two auction regimes. In the entry by patronage auction regime, the firm which offers the highest expected traffic flow on the new link is allowed to add the link. In the entry by generalized price auction regime, the firm which offers the lowest expected generalized price can add the link. Every time a new firm enters, it makes

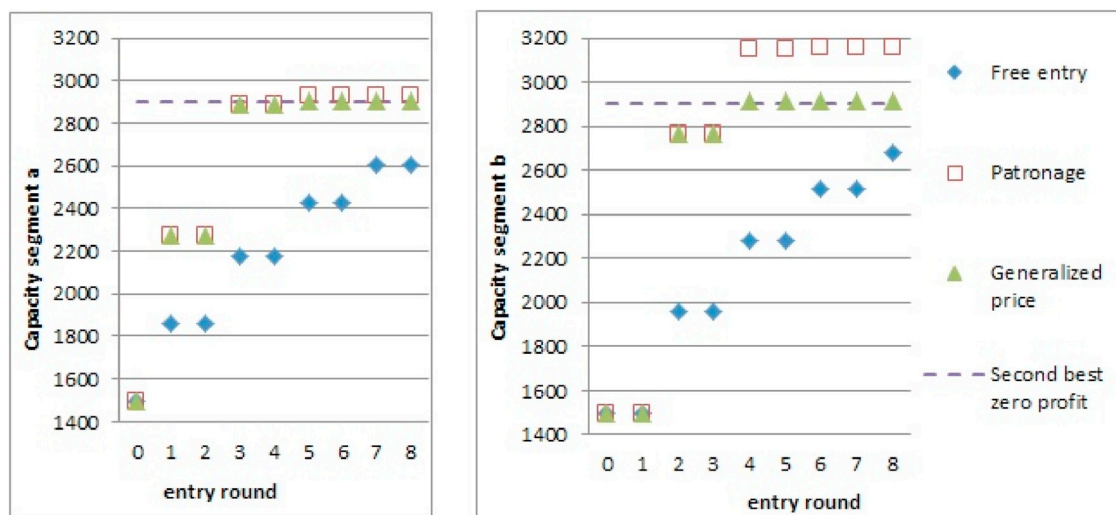


Fig. 2. Development of the capacities of the two links over the entry rounds.

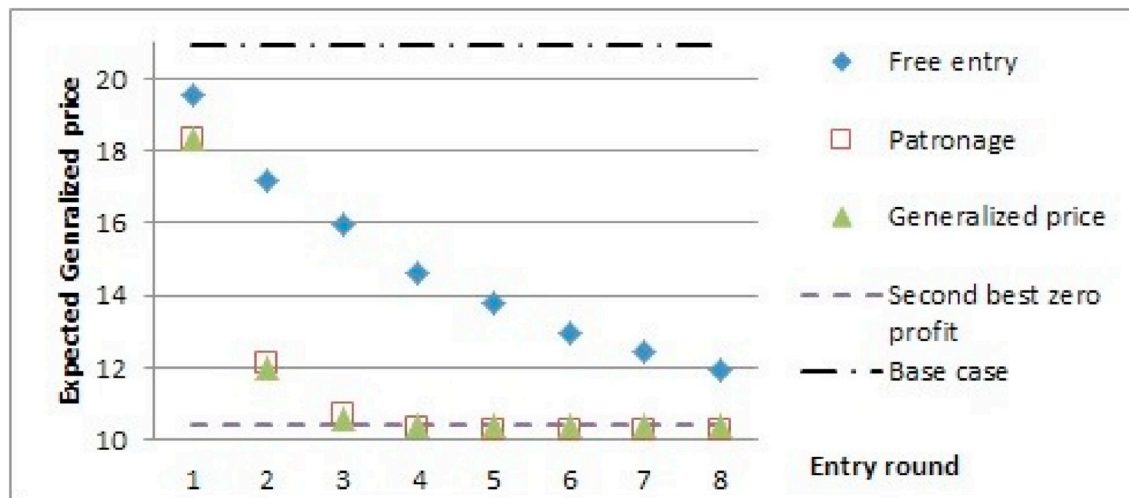


Fig. 3. The generalized price in each entry round.

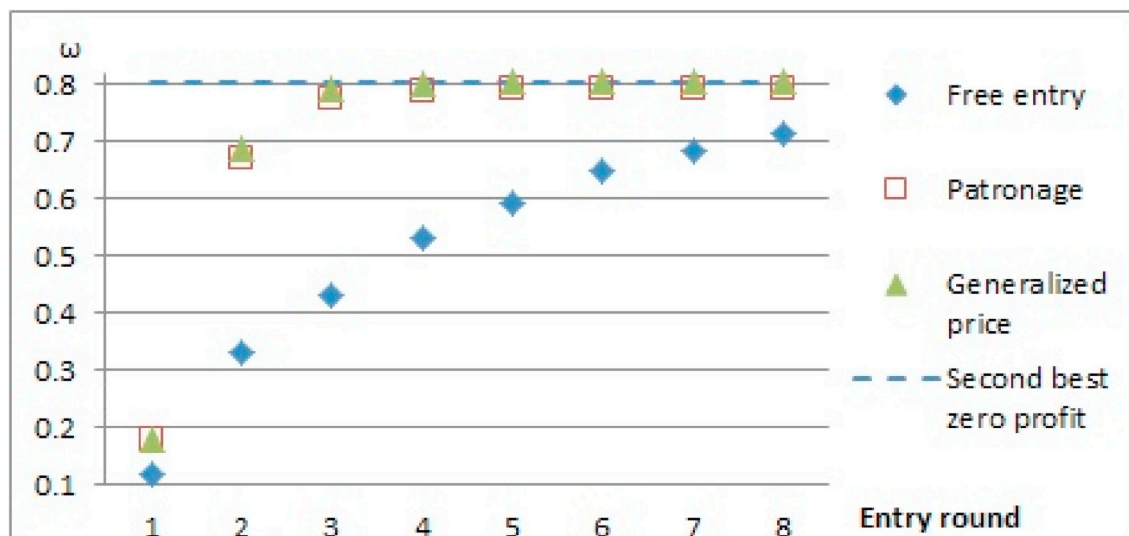


Fig. 4. The Relative efficiency in each entry round.

Note: Relative efficiency is the expected welfare gain relative of a policy from the base case relative to the gain of the first-best social optimum.

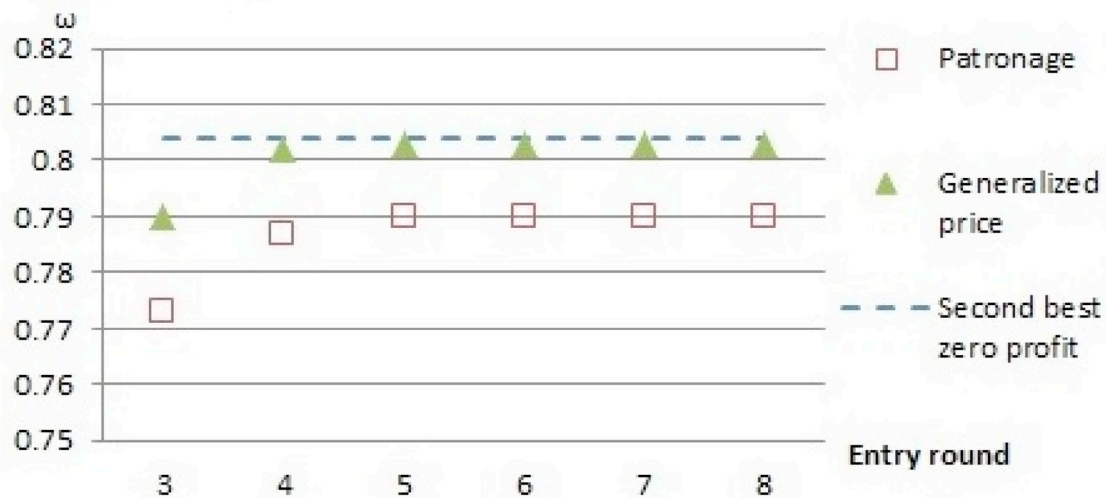


Fig. 5. Zoom of the relative efficiency over the entry rounds for the auction formats.

Note: Relative efficiency is the expected welfare gain of a policy from the base case relative to the gain of the first-best social optimum.

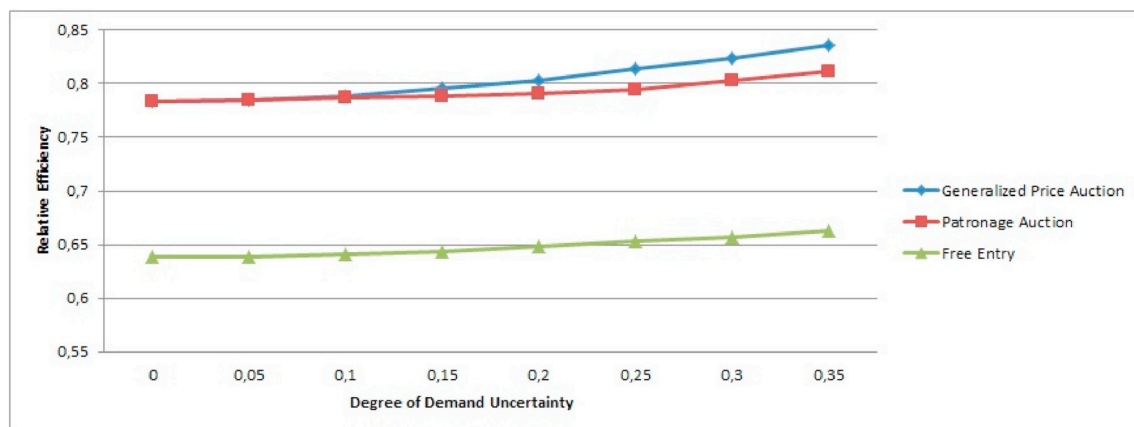


Fig. 6. Sensitivity analysis: Degree of uncertainty.

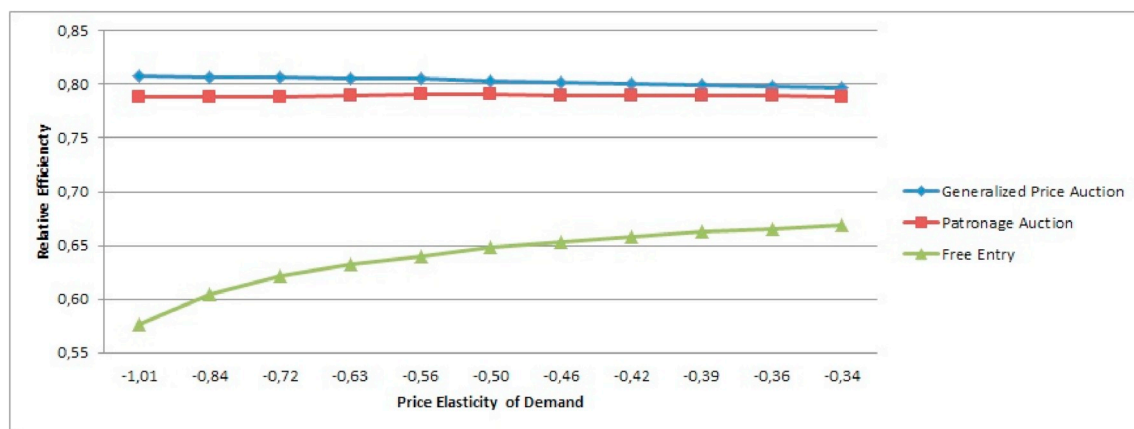


Fig. 7. Sensitivity analysis: Demand elasticity.

zero expected profit since the auction is perfectly competitive. Afterwards, it keeps the toll scheme unchanged. It may not collect enough tolls to cover the capacity cost if later too many firms enter with low tolls. Again, there is no issue with double marginalization and parallel competition remains: first firm 1 enters on segment a , then it enters on segment b , then a new firm 2 enters on segment a , then this same firm 2 enters on b , and so on and on.

Figs. 2–4 show that both auction entry regimes perform much better than free entry: they lead to higher capacities, higher welfare, and lower prices. The development is also much quicker. Already after 4 rounds on entry and 2 firms, the outcome is very similar to the second-best zero-profit case. But due to the assumed sequential nature of entry, capacity will eventually exceed the second-best zero-profit capacities and generalized prices will be lower. This is actually also true with free

entry, but this takes a very long time, and it also occurs with deterministic demand in Verhoef (2008).

The two auction regimes perform rather similarly, and in the above figures it is difficult or even impossible to distinguish the two. Yet, as the zoomed in picture of the development of the relative efficiency in Fig. 5 shows, a generalized price auction consistently performs better than the patronage auction in this numerical model. Whether this is generally true is impossible to say. The figure does clearly show that the two auction formats lead to different results, which is certainly not the case under deterministic demand.

3.3. Sensitivity Analysis

The numerical simulation seems to suggest that entry by either auction performs better than free entry. In addition, entry by the generalized price auction seems to generate higher efficiency than the patronage auction when demand is uncertain. In this section we test the sensitivity of such results with respect to the degree of uncertainty and the price elasticity of demand.

In the numerical simulation, the inverse demand function is represented by $D^i(N^i) = \delta_0^i - \delta_1 \cdot N^i$ and the degree of uncertainty is represented by a , where $\delta_0^h = (1 + a) \cdot \delta_0$ for the high demand state and $\delta_0^l = (1 - a) \cdot \delta_0$ for the low demand state. $a = 0$ means demand is certain, and as a increases, demand becomes more uncertain. $a = 0.2$ is used in the main numerical simulation. Fig. 6 shows the relative efficiency of the three regimes after five rounds of entry, for different degrees of demand uncertainty. For the range considered, the two auctions clearly perform better than free entry, because capacity addition is quicker with auctions. The efficiencies of the two auctions are close, which is consistent with the case without demand uncertainty, where they are equal. Yet, for sufficiently high uncertainty, the generalized price auction clearly leads to a higher welfare than the patronage auction. As the degree of uncertainty increases, the relative efficiency of all three regulatory regimes increases. The main reason is that the optimal capacity increases with the degree of uncertainty, as was also theoretically found for the first-best case.

To study the robustness of the results with respect to demand elasticity, we vary the demand elasticity by changing the intercept and slope of the inverse demand function, keeping the base equilibrium unchanged. For the main simulation in the previous section, the demand elasticity was 0.50. Fig. 7 shows the relative efficiencies over different elasticities. For the parameter range in the simulation, it seems that the two auctions generate similar social welfare, with the generalized price auction performing only slightly better. They both perform much better than free entry, due to their quick addition of capacity. As the demand becomes less elastic, due to larger capacity adjustment under the first-best case and the auctions, the expected social welfare of the three cases increases less significantly than that of the base equilibrium and the free-entry case. As a result, the relative efficiency of the auctions decreases and that of the free-entry case increases.

4. Concluding remarks

This paper has investigated how demand uncertainty influences the efficiency of the private supply of roads in a mixed private/public road network. We have compared different benchmarks and evaluated the efficiency of regulatory policies for both simple static networks and more complex dynamic ones.

Demand uncertainty indeed raises new challenges for policy makers. For simple networks, taking demand uncertainty into consideration, the optimal capacity for the first-best case is larger than its deterministic counterpart. In the second-best zero-profit case, the tolls for serial links are no longer Pigouvian. The patronage and the generalized price auction can no longer achieve the second-best zero-profit result. For more complex networks with dynamic formation of new links, if the

firms with the highest expected profits can add capacity in turn, there is usually over-investment. When we control the process by the patronage auction or the generalized price auction, the expected social welfare increases much more quickly and comes rather close to a steady state after only five rounds. Unlike in the case of deterministic demand, with demand uncertainty the generalized price auction performs better than the patronage auction.

Our findings can be useful for policy makers and practitioners who have to make decisions about road pricing and capacity investment despite prevailing demand uncertainty. For a Pigouvian toll to correct the externality in many markets such as the transport, telecommunications, and energy markets, caution needs to be exercised because demand uncertainty will render some policies less effective in mixed networks. In addition, market competition itself may not be enough to guarantee a quick formation of efficient road networks. Instead, combining the forces of market competition with regulation, especially the generalized price auction, is proven in theory to be a reliable way of providing good public service by private firms. It can form the basis for public-private joint projects in more complex and realistic settings.

In sum, demand uncertainty complicates the evaluation and regulation of the private supply of roads in mixed networks, but clearly in reality there is uncertainty. For future research, we will consider more general networks, dynamic games of capacity addition, user heterogeneity, and optimal auction design.

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